**Domain variables, homogeneity projection, and two notions of entailment**

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**Summary**: The homogeneity effects of plural definites in the restrictors of *every* and *no* are unlike those of plural definites in other (globally) non-monotonic contexts. We claim that this is due to them introducing domain variables that their presupposition and assertion are both predicated of. We also propose that in order to account for NPI licensing facts, the domain variable needs to be dynamically introduced in the existential presupposition. The homogeneity effects of plural definites in the restrictors of these determiners are accounted for in terms of how homogeneity projects through quantifiers, and what notion of entailment is at play.

**Plural definites in non-monotonic contexts**: Plural definites are generally interpreted (quasi-)universally in positive sentences and (quasi-)existentially in negative sentences. In constructions whose meaning can be characterised as the conjunction of positive and negative inferences, they receive 'bipolar readings': they are interpreted (quasi-)universally with respect to the positive part of the meaning and (quasi-)existentially with respect to the negative part of the meaning. Relevant constructions include the scope of *exactly one* in (1), as well as those whose non-monotonicity is due to presupposition such as the complements to the negative factive verb *unaware* in (2) and the aspectual verb *start* in(3). We are not interested in non-maximal readings, and we use definite plurals with numerals so as to block them (e.g. Križ 2016).

(1) Exactly one student read the five abstracts.
  ≈ One student read all five abstracts and no other students read any of them.

(2) The linguist is unaware that her three supervisees are depressed.
  a. Presupposition: All three of the linguists are depressed.
  b. Assertion: The linguist doesn't know of any of them that they are depressed.

(3) The linguist started liking her three supervisees.
  a. Presupposition: The linguist did not like any of her three supervisees before.
  b. Assertion: The linguist now likes all of them.

We observe that the restrictors of *every* and *no*, which trigger existential presuppositions, are exceptions to the above pattern. That is, they are as non-monotonic as the above contexts, due to the existential presupposition and the downward monotonic assertion, but definite plurals do not give rise to bipolar readings there. Rather, they are ambiguous between doubly-universal and doubly-existential readings. To illustrate, consider (4).

(4) Every/No student who took the two introductory courses will take my seminar this term.

We observe that this sentence is ambiguous between the following readings.

(5) *Doubly universal reading of (4)*:
  a. Presupposition: There are students that took both of the introductory courses
  b. Assertion: Every/No student that took both introductory courses will take my seminar this term.

(6) *Doubly existential reading of (4)*:
  a. Presupposition: Some students took one or both of the two introductory courses.
  b. Assertion: Every/No student that took one or both of the introductory courses will take my seminar this term.

Crucially, the bipolar reading, (7), is unavailable.

(7) **Bipolar reading of (4)*:
  a. Presupposition: There are students that took both of the introductory courses
  b. Assertion: Every/No student that took one or both of the introductory courses will take my seminar this term.

For (4), perhaps the doubly universal reading is more prominent. The following examples, however, clearly show that doubly existential readings exist, as their doubly universal readings are implausible given real world knowledge.

(8) a. Everyone who lives in the five Nordic countries hates Russia.
  b. No one who lives in the five Nordic countries loves Russia.

(9) a. Everyone who speaks these three minority languages is over 70.
b. No one who speaks these three minority languages is below 70.

It can be demonstrated that these doubly existential readings are not due to inverse scope by comparing these sentences to the version with an upward monotonic quantifier *someone* [details omitted here]. In what follows we'll focus on *every* to save space.

**Domain variables**: We first account for the doubly universal reading, as well as the lack of a bipolar reading, in the restrictor of *every*. The crucial piece of our analysis is that strong quantificational determiners are associated with a domain variable and their presupposition and assertive meaning are both about it. The denotation of *every* looks like (10) (to be revised).

(10) $[\![\textbf{every}_X]\!]^{w,g} = \lambda P_{(e,t)}.\lambda Q_{(e,t)}: \; g(X) \neq \varnothing \wedge \forall x \in g(X)[P(x)].\; \forall x \in g(X) Q(x)$

Importantly, the restrictor denotation $P$ is evaluated only in the existential presupposition, which is a positive context. Consequently, when the restrictor contains a plural definite, it will receive a (quasi-)universal reading. The assertion inherits this (quasi-)universal reading by making reference to the subset $g(X)$ of $P$ rather than $P$ itself. The doubly universal reading for (4) is thus paraphrasable as 'There is a set of students that took both courses and all of those will take my seminar'. This analysis can be given a more precise formulation by adopting Križ's 2016 trivalent theory for homogeneity and making both presupposition and assertion trivalent statements, but we refrain from presenting the formal details here for reasons of space.

On the other hand, for other non-monotonic contexts, the presupposition and assertion do not have a shared component. To illustrate, consider the denotation of *unaware*:

(11) $[\![\textbf{unaware}]\!]^{w,g} = \lambda p_{(s,t)}.\lambda x_e: \; p(w).\; \forall w' \in \text{Dox}_{w,x}[\neg p(w')]$

Crucially the complement denotation $p$ is evaluated once in the factive presupposition, which is positive, and once in the assertion, which is negative. Consequently, if the complement clause contains a plural definite, it receives a (quasi-)universal reading in the presupposition and an (quasi-)existential reading in the assertion, which is a bipolar reading.

**Dynamic Strawson monotonicity**: There is, however, an issue with the denotation in (10). (12a) with *American* only differs from (12a) without *American* in the presupposition. With the presuppositions true, there is equivalence in the assertions. This means that *every* in (10) is technically both Strawson upward and downward monotonic on its restrictor.

(12) a. Every (American) student who solved any problem is happy.
b. *Most students who solved any problem are happy.

In fact, when generalised, all quantificational determiners will be both Strawson upward and downward monotonic. Such environments do not license weak NPIs and minimisers (Lahiri 2002, Guerzoni & Sharvit 2007). So according to (10), (12a) should be as degraded as (12b).

A solution to this problem comes about if we dynamicise the analysis. Specifically, we propose that the presupposition of *every* introduces the domain variable via the dynamic existential quantifier, and its assertion anaphorically refers back to it. We can formalise this idea by combining dynamic semantics and Križ's trivalent semantics for homogeneity in a bidimensional trilateral system, but the space limitation prevents us from giving the formal details of this idea in full in this abstract. However, here is the essence of the analysis. The presupposition of $\ulcorner \textbf{every}^X(P(X))(Q(X)) \urcorner$ is a trivalent proposition that is true when there is a non-empty set $X$ that is identical to the maximal set of (contextually relevant) $P$-individuals. The assertion of this sentence is also a trivalent proposition and is true if every atomic member of $X$ is a $Q$-individual (assuming distributivity). The crucial assumption here is that the dynamic presupposition updates the context (cf. Beaver 1992, Elliott & Sudo 2020), so that the domain variable $X$ that is dynamically introduced in the presupposition can be referenced in the assertion.

This is a dynamic version of (10), but crucially, this system allows for a dynamic version of Strawson entailment, as in (13), with the idea being that NPI-licensing is subject to (13).

(13) A dynamic statement $\phi$ dynamically Strawson entails another dynamic statement $\psi$ iff for each context $c$ such that the presuppositions of $\phi$ and $\psi$ are satisfied, whenever $c$ updated

with $\phi$ is non-empty, then $c$ updated with $\psi$ is also non-empty.

This will correctly render *every* dynamically Strawson downward monotonic in (12a) thereby licensing the NPI: in every $c$ with a set $Y$ being the American students who solved a problem there is a set $X$ being the students overall who solved a problem. Clearly, if update of $c$ with the information that every $x \in X$ is happy leads to a non-empty context, so does update of $c$ with the information that every $y \in Y$ is happy. *most*, however, is dynamically Strawson non-monotonic.

**Homogeneity projection**: Lastly, we propose that the ambiguity between the doubly universal and the doubly existential reading stems from the way homogeneity projects through quantifiers.

Working in a static system without presuppositions, Križ 2015 proposes that homogeneity projects systematically through quantifiers according to the following 'supervaluation' rule. By assumption, quantificational determiners have bivalent meanings, but each of them comes in two versions: one version turns its arguments into bivalent predicates by collapsing falsity and homogeneity failure; the other version by collapsing truth and homogeneity failure. A quantified statement $\ulcorner D(P)(Q) \urcorner$ is true if both versions of the determiner $D$ make the statement true, false if both make the statement false, and a homogeneity failure otherwise.

We follow Križ's spirit but alter the projection rule so it can be relativised to different notions of entailment. The idea is that we order the possible bivalent determiner denotations in terms of a given notion of entailment, and pick the supremum in the space of possible determiner denotations. In our bidimensional trivalent dynamic system, each strong determiner has two possible bivalent denotations for its presupposition and two possible bivalent denotations for its assertion, so for each determiner, there are four combinations of a presuppositional denotation and an assertive denotation. In the case of *every*, the two presuppositional denotations $(\!(\textbf{every})\!)$ and the two assertive denotations $[\![\textbf{every}]\!]$ look as follows. To simplify, the assertion is made externally static.

$$c(\!(\textbf{every}^X\ \phi\ \psi)\!)^{\#\mapsto 0}$$
$$= \left\{ \langle w, g^{A/X} \rangle \mid \langle w, g \rangle \in c, A = \left\{ a \mid \exists g'(\langle w, g' \rangle \in c(\!(\phi)\!)^+[\![\phi]\!]^+ \wedge g'(X) = a) \right\} \neq \varnothing \right\}$$
$$c(\!(\textbf{every}^X\ \phi\ \psi)\!)^{\#\mapsto 1}$$
$$= \left\{ \langle w, g^{A/X} \rangle \;\middle|\; \langle w, g \rangle \in c, A = \left\{ a \;\middle|\; \exists g'(\langle w, g' \rangle \in \bigcup_{*,\star \in \{\#,+\}} c(\!(\phi)\!)^*[\![\phi]\!]^\star \wedge g'(X) = a) \right\} \neq \varnothing \right\}$$

$$c[\![\textbf{every}^X\ \phi\ \psi]\!]^{\#\mapsto 0} = \left\{ \langle w, g \rangle \in c \mid g(X) \subseteq \left\{ a \mid \langle w, g^{a/X} \rangle \in c(\!(\psi)\!)^+[\![\psi]\!]^+ \right\} \right\}$$
$$c[\![\textbf{every}^X\ \phi\ \psi]\!]^{\#\mapsto 1} = \left\{ \langle w, g \rangle \in c \;\middle|\; g(X) \subseteq \left\{ a \;\middle|\; \langle w, g^{a/X} \rangle \in \bigcup_{*,\star \in \{\#,+\}} c(\!(\psi)\!)^*[\![\psi]\!]^\star \right\} \right\}$$

Crucially, there are two ways of partially ordering the four pairs of a presuppositional denotation and an assertive denotation, corresponding to two notions of entailment. The plain entailment will order the presuppositional dimension and the assertive dimension separately, while dynamic Strawson entailment orders bidimensional denotations as pairs directly. More specifically, let $\Rightarrow$ be generalised entailment and $\overset{\text{DS}}{\Rightarrow}$ be generalised dynamic Strawson entailment.

(14) a. $\langle \pi_1, \alpha_1 \rangle \Rightarrow \langle \pi_2, \alpha_2 \rangle$ iff $\pi_1 \Rightarrow \pi_2 \wedge \alpha_1 \Rightarrow \alpha_2$

b. $\langle \pi_1, \alpha_1 \rangle \overset{\text{DS}}{\Rightarrow} \langle \pi_2, \alpha_2 \rangle$ iff $\forall c \forall (w, g) \in c[[\exists (w, g') \in c[\pi_1] \wedge g \leqslant g' \wedge \exists (w, g'') \in c[\pi_2] \wedge g \leqslant g''] \rightarrow [c[\pi_1][\alpha_1] \neq \varnothing \wedge c[\pi_2][\alpha_2] \neq \varnothing]]$

It can be demonstrated that in the case of *every* whose restrictor contains a plural definite, homogeneity projection with plain entailment $\Rightarrow$ will yield the doubly universal reading, while homogeneity projection with dynamic Strawson entailment $\overset{\text{DS}}{\Rightarrow}$ will yield the doubly existential reading.